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A study of three-dimensional natural convection in high-pressure mercury lamps—I. Parametric variations with horizontal mounting

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Abstract—A three-dimensional numerical model utilizing curvilinear coordinates and an efficient solution method has been used to investigate natural convection in horizontal, high-pressure mercury-vapor arctubes. For horizontal arcs, convection velocities are predominantly transverse to the arc axis. The associated upward bowing of the arc column results in non-uniform heating of the quartz wall which can substantially degrade the performance of the modern horizontal metal halide lamp. This investigation shows how the design parameters of the arctube, such as the mercury pressure, the curvature of the curved arctube, the offset electrode distance, and the electrode insertion length affect the temperature distribution in the arctube. Qualitatively, a downward curved arctube can move the hot arc core downward, and offset electrodes can improve temperatures near the ends of the arctube. The evolution of the design improvement of a 400 W high-pressure mercury lamp with the aim of centering the arc and having isotherms conform to the wall is achieved through better quantitative understanding of the transport characteristics.

1. INTRODUCTION

NATURAL convection plays a significant role in the physics of the high-pressure mercury discharge. Convection influences the stability and location of the discharge, and transports energy away from the discharge. As is evidenced in refs. [1–4], little detailed knowledge has been accumulated in understanding the flow pattern occurring in a horizontal discharge, despite pronounced progress that has been made in other areas, such as chemical aspects.

In this investigation, we are interested in horizontal arcs where convection velocities are predominantly transverse to the arc axis. The associated upward bowing of the arc column results in non-uniform heating of the confining quartz wall. The excess heating at the top may soften and bulge out the quartz; the bottom wall temperatures may be too low to provide adequate metal halide vapor pressure for mercury arcs with metal halide additives. Two methods have been considered to avoid this problem. One is to apply transverse magnetic fields [4] and the other is to bend the arctube. The latter is the approach adopted in this investigation.

Since life and performance of high-pressure discharge lamps are determined by their temperature profiles, understanding the effect of design parameters of the arctube (such as the mercury pressure, the curvature of the curved arctube, the electrode offset, and the like) on the temperature distribution within the discharge volume, is critical. By understanding the characteristics associated with each of these parameters, the hot arc core may be centered, with isotherms conforming to the quartz wall. Consequently, the temperature difference between the top and bottom wall can be minimized.

Here we report a study of a three-dimensional natural convection process by numerically solving the coupled momentum, mass continuity, energy, state, and electric field equations on the curvilinear coordinate system. The numerical model has been applied to study the effects of the various design parameters on the convection and temperature distributions inside the arctube. The results will be appraised against the experimental observations. The objectives of this investigation are: to help interpret and understand observed experimental phenomena; to guide the design improvement of hardware experiments; and to help establish the influence and sensitivity of individual design parameters on the lamp performance. Among the various design parameters, it will be demonstrated that curving the arctube and offsetting the electrodes are the most important factors in obtaining a desirable temperature distribution within the arctube. By presenting a large number of carefully assembled results, a good assessment can be made in terms of the capability of the present computational model. Quantitative information obtained from this model has greatly improved our understanding and design of the modern horizontal metal halide discharge lamp.

2. NUMERICAL ALGORITHM AND BOUNDARY CONDITIONS

The model provides a solution to the coupled discharge equations—the energy, mass continuity, and momentum equations for the discharge fluid, a simplified treatment of radiation, and the electric field equations. These equations are solved using finite volume techniques and the method of successive approximations. The discharge medium is assumed to exhibit local thermodynamic equilibrium, and to be steady in time. Inputs to the model include the arctube geometry, the orientation of the arctube relative to gravity, and the electrical power input. The self-consistent solution includes the fluid temperature and velocity, the electrostatic potential, and the electric current density, all of which are determined at each node in the staggered three-dimensional grid. The temperature-dependent fluid properties are taken from Zollweg [3] for the case of a 3 atm discharge in mercury. For variations in mercury pressure, the mercury density is taken to be linearly proportional to the mercury pressure.

Figures 1(a) and (b) show a typical 400 W commercial mercury lamp and its arctube, respectively. The computational model geometry and grid distribution of a straight arctube appear in Figs. 1(c) and (d). The fused quartz arctube is characterized by a cylindrical center section of constant bore, which is capped by bowl shaped end chambers. The electrodes protrude into the arctube by a distance called the insertion length and offset electrode positions can be specified. The calculations are mainly carried out for a 400 W mercury discharge.

To resolve the geometrical complexities of the arctube and electrodes, the transport equations are solved by an algorithm developed for solving Navier–Stokes flows using general non-orthogonal curvilinear coordinates [5–7]. The volumetric source terms driving the flow are ohmic heating and radiation cooling, which enter the energy equation, and gravitation, which enters the momentum equations.

The governing equations are first written in the strong conservation law form in Cartesian coordinates for the dependent variable ϕ

$$\frac{\partial}{\partial x}(\rho u\phi)+\frac{\partial}{\partial y}(\rho v\phi)+\frac{\partial}{\partial z}(\rho w\phi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma\frac{\partial\phi}{\partial z}\right)+R(x,y,z). \quad (1)$$

Here, Γ is the effective diffusion coefficient and R the source term, including the gravitational force. Equation (1) can represent the continuity, momentum, energy and electric field equations (equation (2)). When new independent variables ξ, η, and γ are introduced, equation (1) changes according to the general transformation $\xi=\xi(x,y,z)$, $\eta=\eta(x,y,z)$, $\gamma=\gamma(x,y,z)$. The result of this coordinate transformation is to transform the arbitrarily shaped physical domain into a rectangular parallelepiped.

The electric field equations can be expressed in terms of electrostatic potential, which takes the form

$$\nabla\cdot\sigma\nabla\Phi=0. \quad (2)$$

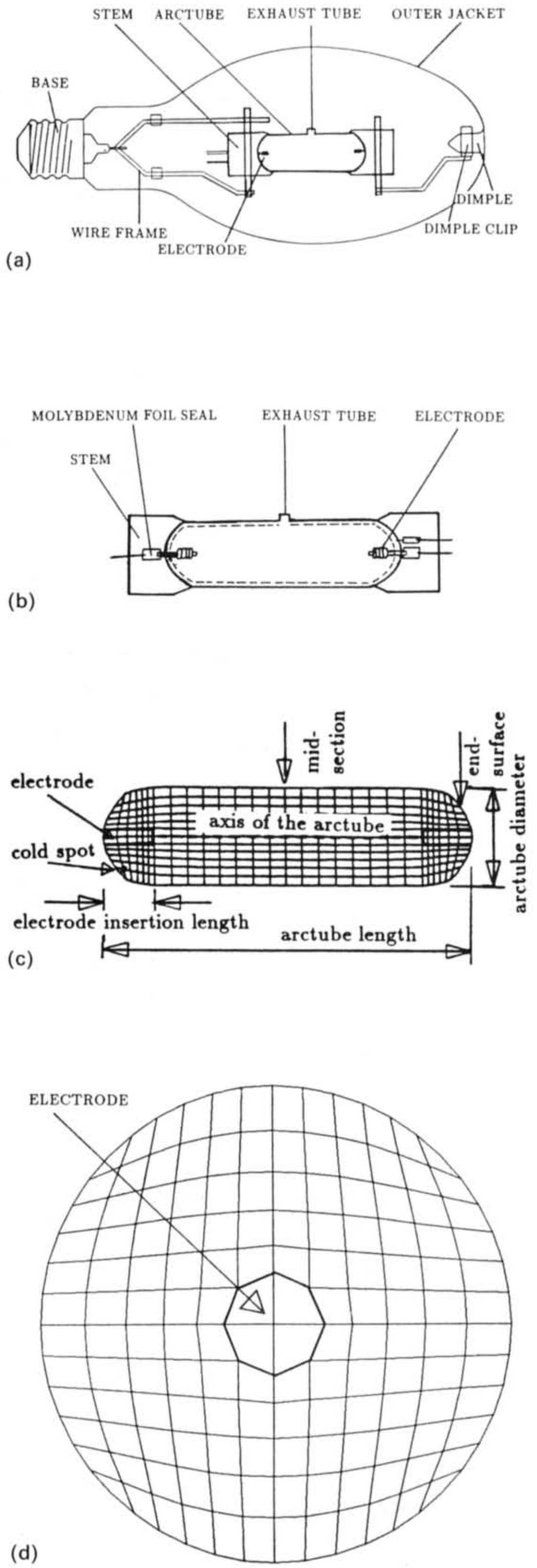


FIG. 1. Schematic drawing of a 400 W mercury lamp, geometry, and grid system: (a) schematic drawing of a 400 W mercury lamp; (b) schematic drawing of a 400 W mercury arctube; (c) side-view (central plane) of arctube, grid (29 × 13); (d) grid (13 × 13) in the bowl-end surface.

Here, σ is the electrical conductivity, and Φ the electrostatic potential. It should be noted that the strong temperature dependence of the electrical conductivity [3] produces a strong coupling between the electric field equations and the fluid equations. This is especially true near the electrode tips, where this coupling is responsible for attaching the arc to the electrode tips. The full treatment of gas to gas radiation is very complex and is not practical for three-dimensional problems because the necessary computations would consume enormous computer time. The simplified treatment is given by

$$\text{radiant heat loss} = A_{\text{urad}} + 1.425 \times 10^{-14} \exp\,[C/T] \tag{3}$$

where $C = -8.0661 \times 10^4$; T is the temperature; and A_{urad} a constant which represents the energy (per unit volume) absorbed in the cold gas region.

Boundary conditions are imposed at the quartz wall of the arctube and surfaces of the electrodes. For the fluid flow calculation, conventional no-slip boundary conditions are applied on the entire boundary for the velocity components, an isotherm is specified at the quartz wall for temperature, and a linear temperature profile is specified along the electrodes. For electric field calculations, the electrodes have surfaces of uniform electrostatic potential, and the normal component of the current density is zero at the wall. The convective flow in the discharge is assumed to be purely laminar; this assumption is supported by both the experimental evidence and the calculated Grashof number. For all the derivatives in the transport equations, convection terms included, the second-order central differencing scheme is employed for dis-

Table 1. Model inputs and outputs

Parameters	Straight arctube
Model inputs:	
mercury pressure (atm)	3
input power (W)	400
arctube diameter (mm)	14.78
arctube length (mm)	55.61
electrode insertion length (mm)	7.64
offset electrode distance (mm)	0
radius of bend (mm)	infinite
angle of bend (deg)	0
Model outputs:	
core displacement from axis	2.1 mm
maximum temperature in mid-section	6328 K
maximum velocity in mid-section	10.3 cm s^{-1}

Note. Mercury pressure and input power are considered as lamp operating parameters. The rest of the input data are all related to arctube geometry. Core displacement, defined as the distance between the center of the hot core and the center of the mid-section, is positive if the core is above the axis, and is negative if it is below the axis. Offset electrode distance is defined as the distance between the axis of the arctube and the axis of the electrode.

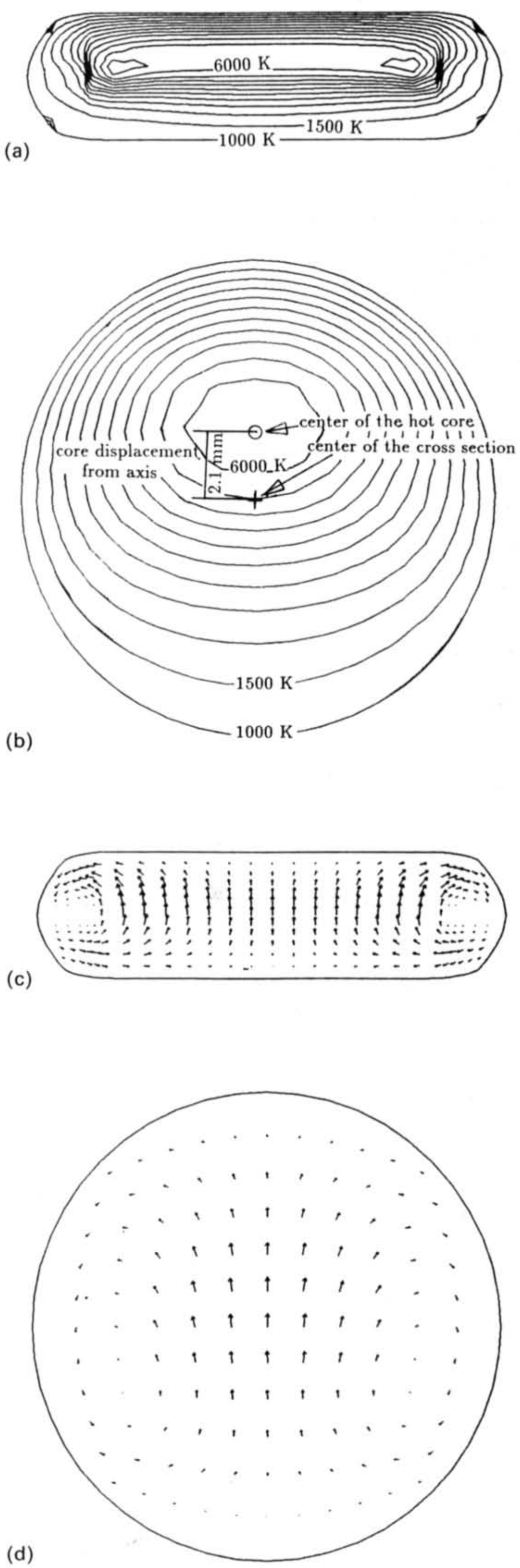


FIG. 2. Temperature contours and convection patterns: (a) temperature contours in central plane (contour interval is 500 K); (b) temperature contours in mid-section (contour interval is 500 K); (c) convection pattern in central plane; (d) convection pattern in mid-section.

cretization. A multigrid method [8] is employed for solving the resulting difference equations.

To handle the imposed geometrical constraints and to maintain a flexible grid distribution, a three-dimensional grid generation code has been developed for the arctube. The algorithms use a zonal approach to account for the various geometrical components in the arctube, e.g. electrodes, middle section, and bowl-end sections. These various components are then patched together by solving a system of elliptic partial differential equations with multiple outer and inner boundaries. Representative views of the grid for the 400 W arctube are shown in Fig. 1. A three-dimensional model is essential to capture the physical phenomena in the geometries considered in this investigation. In the present work, a series of calculations has been conducted with different number of nodes, ranging from $29\times13\times13$ to $29\times29\times29$, the solutions were found to be within 0.1% difference in the predicted gas temperature field.

3. SPECIFICATION AND FLOW FIELD OF BASE GEOMETRY

Table 1 shows the base geometry and input data for the arctubes used in this investigation. The input power and mercury pressure are 400 W and 3 atm, respectively. The base geometry is a straight arctube. Figure 1 shows the side-view and end surface of the arctube. For horizontal operation, there are two symmetrical planes. One is identified as the *central plane* (Fig. 1(c)), and the other is identified as the *mid-section*. Temperature contours and velocity vectors are shown in these two planes. Table 1 also contains some key outputs from the model.

Figures 2(a) and (b) show temperature contours in the central plane and in the mid-section with a maximum temperature of 6328 K. It is noted that the hot arc column bows upward, the arc core lies closer to the top wall, the isotherms do not conform to the bottom wall, and the temperature gradient is not uniform. Figures 2(c) and (d) show the convection velocity patterns. The length of the arrows is proportional to the velocity, with a maximum velocity of 10.3 cm s^{-1}. Both the temperature contour and the flow pattern in the mid-section are generally consistent with those computed for a 2.89 atm mercury arc with no transverse magnetic field in ref. [4].

4. PARAMETRIC STUDY OF HORIZONTAL ARCTUBES

The following parametric study of a horizontal arctube is carried out primarily to understand the trends in temperature distributions as a result of varying design parameters, such as mercury pressure (density), input power, and so on. It also reveals sensitivities to these design parameters. Parameters studied here are varied with respect to the model input data in Table 1. Some representative cases are highlighted here.

4.1. *Straight arctube*

Figure 3 shows the effects of gas pressure on the natural convection pattern as well as on temperature. Several observations can be made. The hot core moves upward and the maximum temperature in the mid-section increases with increasing mercury pressure. Consequently, the temperature patterns are more nonuniform with higher mercury pressure; this results in degradation of the quality of light as well as the expected life of the lamp. This phenomenon is mainly due to the fact that the Grashof number is higher with higher mercury pressure (and hence higher density), which creates stronger natural convection. This trend is similar to the experimental observations made on horizontal metal-halide lamps.

Figure 4 shows the effects of arctube diameter on the temperature patterns. The hot core appears to move upward continuously with increasing arctube diameter. This phenomenon is also due to the change of the Grashof number, which is proportional to the characteristic length (arctube diameter) to the third power. The arc core also becomes constricted with increasing arctube diameter. Thus, reducing arctube diameter would improve the isotherm distribution, and improve temperature uniformity in the wall region.

4.2. *Curved arctube*

Next, the case of a curved arctube is investigated. The specification of the base condition here is identical to that shown in Table 1, except that the radius of bend is 25.4 mm and the angle of bend is 40°. Figure 5 shows both the side view and grid distribution as well as the calculated velocity and temperature fields. Compared to the straight arctube (Fig. 2), the hot arc column now lies more along the axis, the arc core lies more at the center of the mid-section, the isotherms conform more to the wall except near the ends of the arctube, and the temperature gradient is much more uniform.

Parametric studies have also been conducted for the curved arctube. Similar results are obtained. For example, by varying the mercury pressure, it is found that the hot core continuously moves upward with increasing mercury pressure for both straight and curved arctubes, although the amount of core displacement is much larger with the straight arctube than with the curved arctube. Figure 6 shows the isotherm plots for the curved case. The maximum temperature also increases with increasing mercury pressure for the curved arctube, but the rate of increasing temperature is much larger with the straight arctube than with the curved arctube. Overall, the isotherms move upward with increasing mercury pressure, hence the isotherms do not conform to the lower portion of the curved arctube as the mercury pressure increases.

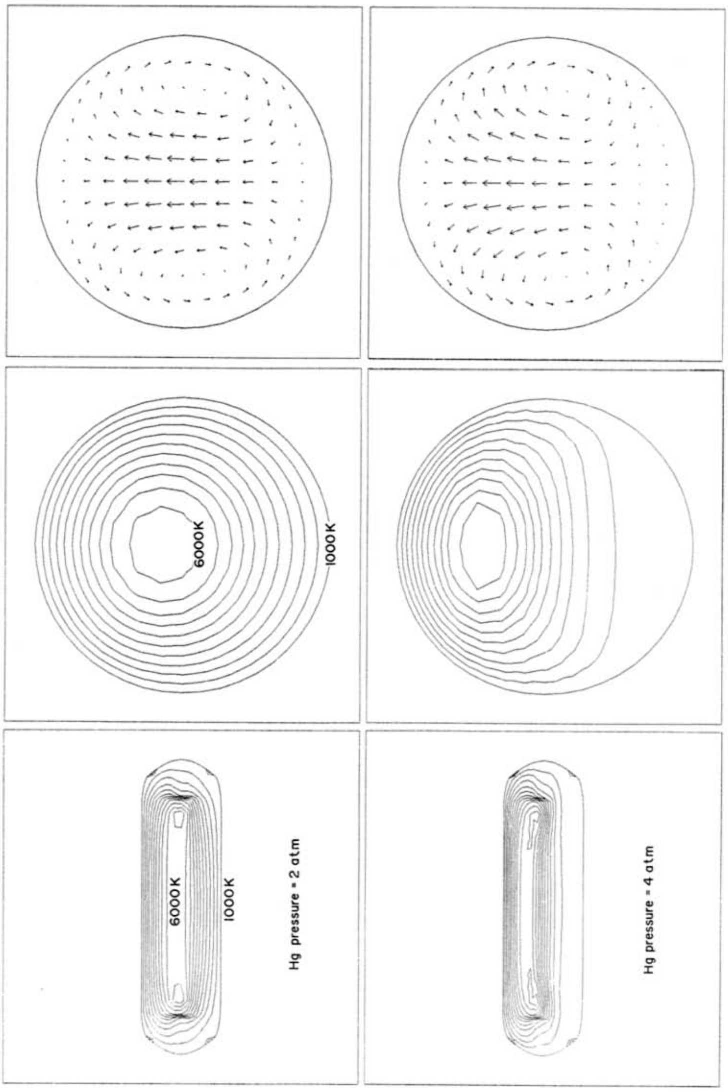


FIG. 3. Effects of varying mercury pressures: temperature contours and velocity field in central plane and mid-section.

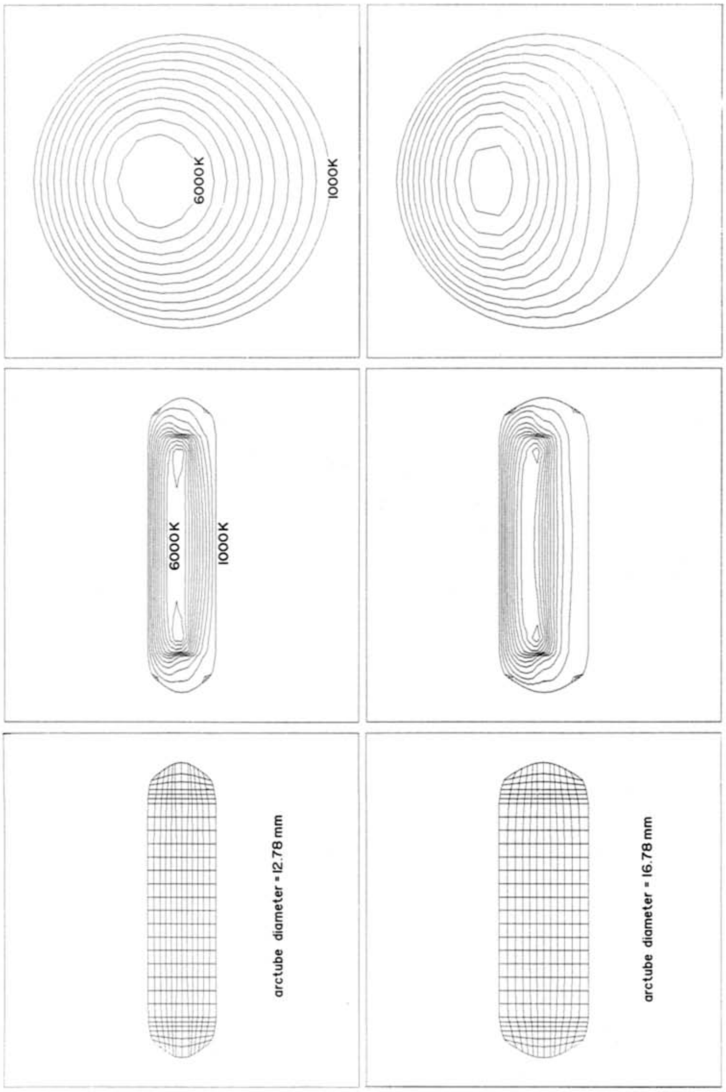


FIG. 4. Geometry of straight arctubes and effects of varying arctube diameters: temperature contours in central plane and mid-section.

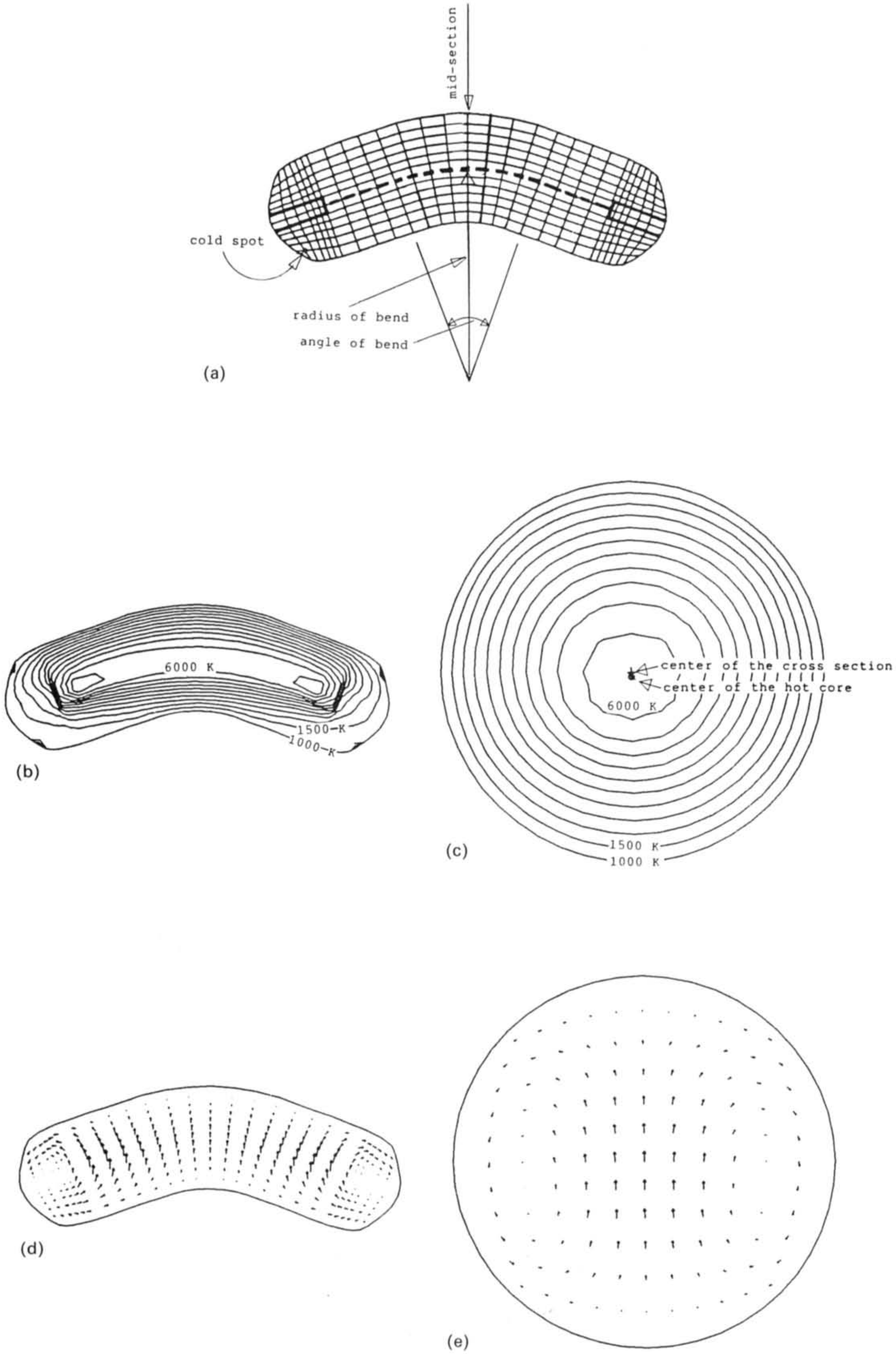


FIG. 5. Geometry of a curved arctube and its temperature contours and convection patterns: (a) geometry of a curved arctube (central plane), grid (29 × 13); (b) temperature contours in central plane (contour interval is 500 K); (c) temperature contours in mid-section (contour interval is 500 K); (d) convection pattern in central plane; (e) convection pattern in mid-section.

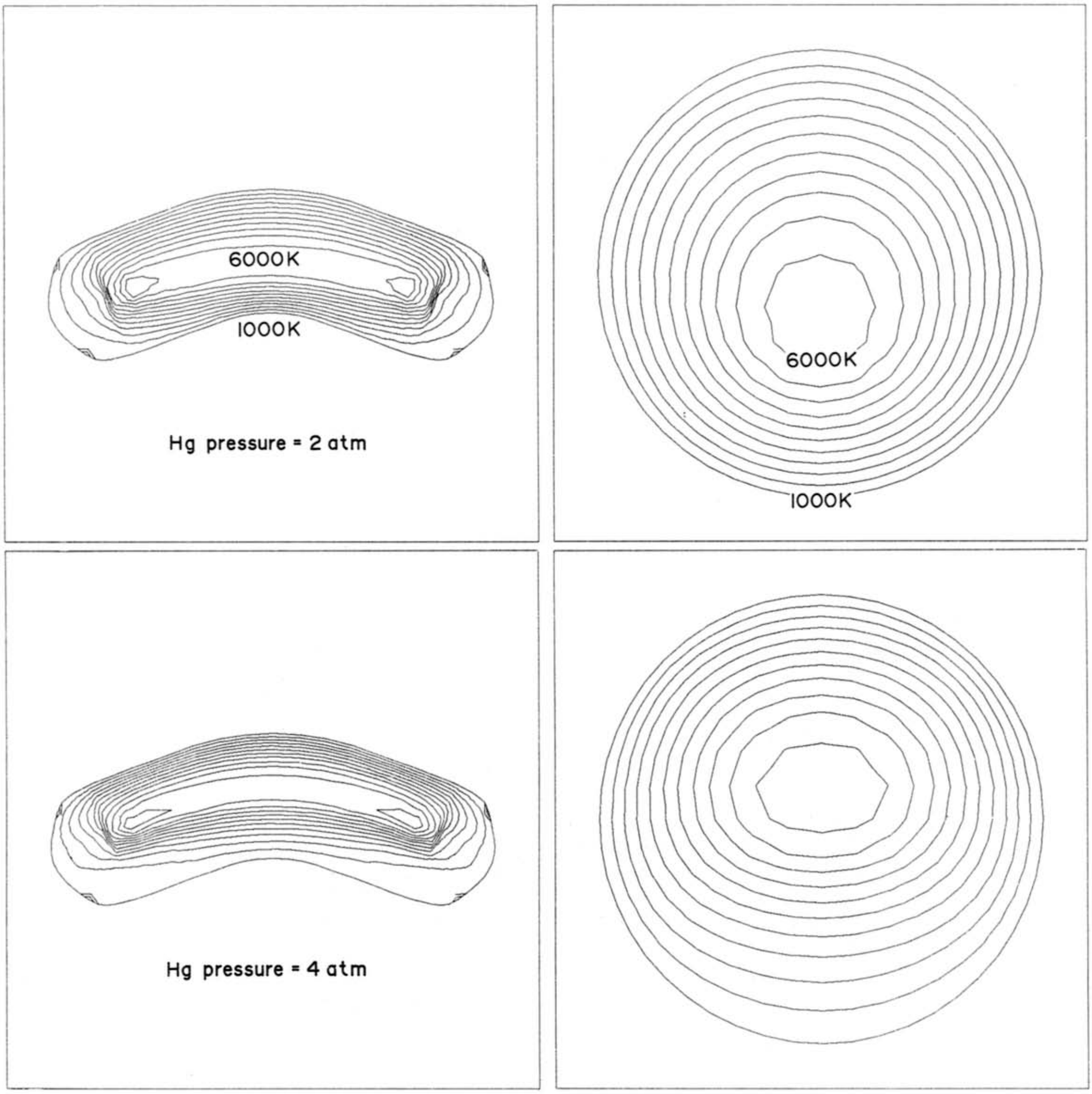


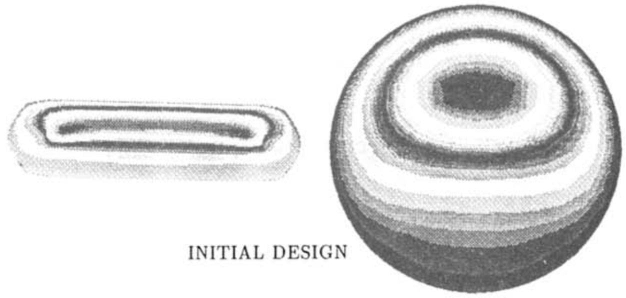
FIG. 6. Effects of varying mercury pressures: temperature contours in central plane and mid-section.

4.3. *Important observations and experimental verification*

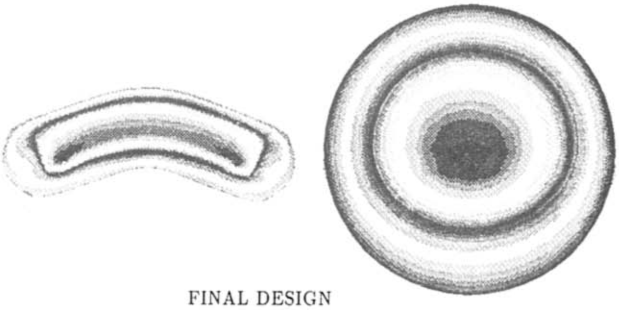
Based on a series of computations, it is found that the design parameters can be ranked *in order of their influence*.

(1) Curvature of the arctube (defined by radius of bend and angle of bend).
(2) Offset electrode distance.
(3) Arctube diameter.
(4) Electrode insertion length.

With understanding the characteristics associated with each of these factors, a case is presented to show that it is theoretically possible to achieve an optimal geometry under a prespecified operating condition. In this investigation, an optimal geometry is defined by producing a centered arc with isotherms conforming to the arctube wall. Based on the straight arctube as the initial design, modifications have been made according to the above prioritized items of design parameters. A qualitative assessment is made in Figs. 7 and 8, where the temperature characteristics obtained from the photographs and numerical computations are shown for both the initial straight and the subsequently optimized curved arctubes. The upward bowing of the high-temperature core in the horizontal arctube and the greatly improved uniformity of temperature distribution resulting from the downward bending of the arctube are consistently contrasted by both the theoretical predictions and experimental observations.



INITIAL DESIGN



FINAL DESIGN

FIG. 7. Temperature distribution in both the initial and final designs of arctube.

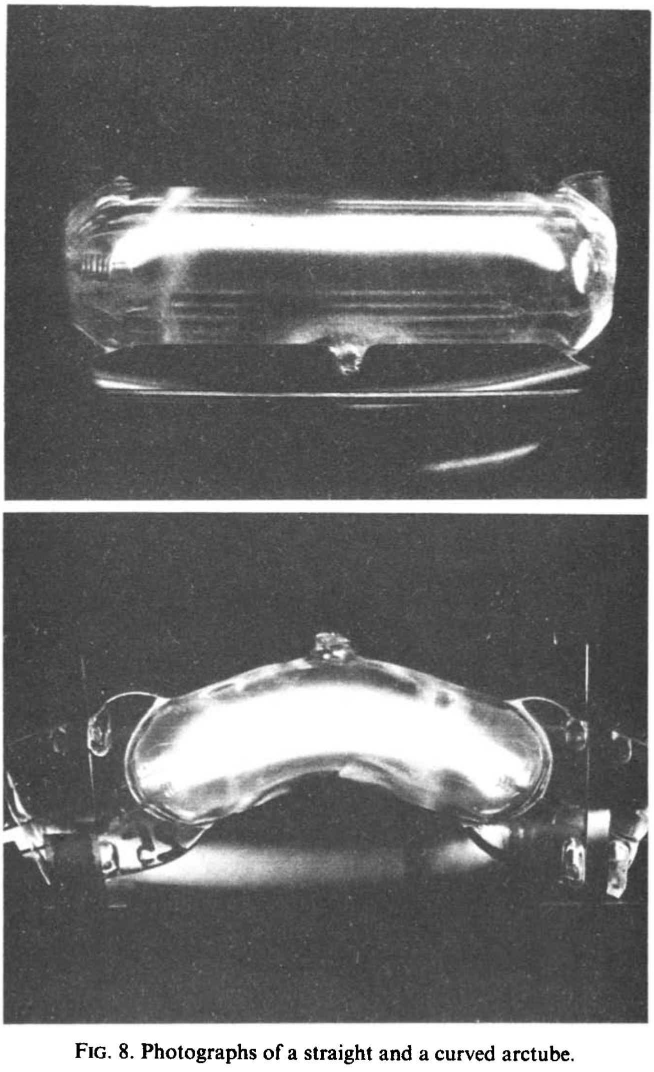


FIG. 8. Photographs of a straight and a curved arctube.

5. SUMMARY AND CONCLUDING REMARKS

A series of three-dimensional calculations has been performed for the natural convection processes in a horizontal high-pressure mercury lamp. To improve the quality of light and the life span of discharge lamps, it is very desirable to center the discharge on the arctube axis and have a uniform temperature distribution in the wall region. Many parametric variations are conducted to identify the important design parameters. The major findings are summarized here.

(1) Computer results show the upward arc bowing within a straight horizontal arctube.

(2) The temperature and convection patterns of a straight arctube agree with those in ref. [4], in which a two-dimensional computer model was employed.

(3) Curved arctubes have much more evenly distributed temperature patterns than the straight tube. The hot core of the arc moves downward with downward body curvature of the arctube.

(4) A good combination of curving the arctube and offsetting the electrodes can effectively improve the temperature distribution within the arctube and consequently improve the performance of the lamp.

(5) The hot arc locations predicted by the present model are consistent with experimental observations.

This investigation has emphasized the influence of geometry as well as operating conditions on the temperature distribution. The example of a 400 W high-pressure mercury arc is used to demonstrate the usefulness of the findings. A prioritized list of parameters has been identified to develop an optimized design. The advanced computational capabilities offered by the present numerical model have made drastic improvement resulting from the parameteric optimization and better understanding of transport mechanisms occurring in the present lamp possible.

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ETUDE DE LA CONVECTION NATURELLE TRIDIMENSIONNELLE DANS LES LAMPES A MERCURE A HAUTE PRESSION—I. VARIATIONS PARAMETRIQUES AVEC MONTAGE HORIZONTAL

Résumé—Un modéle numérique tridimensionnel utilisant des coordonnées curvilignes et une méthode efficace de résolution traite la convection naturelle dans les tubes à arc horizontaux avec vapeur de mercure à haute pression. Pour les arcs horizontaux, les vitesses convectives sont principalement transversales à l'axe de l'arc. La courbure vers le haut de l'arc, conduisant au chauffage non uniforme de la paroi de quartz, peut dégrader les performances des lampes halogènes modernes horizontales. Cette étude montre comment les paramètres du tube d'arc, tels, que la pression du mercure, la courbure du tube d'arc, la distance de l'électrode et la longeur d'insertion de l'électrode, affectent la distribution de température dans le tube d'arc. La conception d'une lampe de 400 W à mercure à haute pression, avec centrage de l'arc et avec des isothermes correctes à la paroi, permet une meilleure compréhension quantitative des caractéristiques du transfert thermique.

UNTERSUCHUNG DER DREIDIMENSIONALEN NATÜRLICHEN KONVEKTION IN HOCHDRUCK-QUECKSILBERDAMPFLAMPEN—I. PARAMETERUNTERSUCHUNGEN BEI WAAGERECHTER MONTAGE

Zusammenfassung—Ein dreidimensionales numerisches Modell wird verwendet, um die natürliche Konvektion in waagerechten Hochdruck-Quecksilberdampflampen zu untersuchen. Das Modell benutzt gekrümmte Koordinaten und eine effiziente Lösungsmethode. Bei waagerechtem Lichtbogen verläuft die Konvektion vorherrschend quer zur Lichtbogenachse. Damit verbunden ist eine Aufwärtskrümmung der Lichtbogensäule, die zu einer ungleichmäßigen Beheizung der Quarzwand führt; dadurch kann die Leistungsfähigkeit der modernen waagerechten Metallhalogenlampen leiden. Die Untersuchung zeigt, wie die Konstruktionsparameter der Bogenröhre wie Quecksilberdruck, Krümmung der Bogenröhre, Elektrodenabstand und Eintauchtiefe der Elektroden die Temperaturverteilung in der Bogenröhre beeinflussen. Qualitativ läßt sich sagen, daß bei einer nach unten gekrümmten Bogenröhre der heiße Kern des Lichtbogens nach unten wandert und daß abgesetzte Elektroden die Temperaturen an den Enden der Röhre günstiger gestalten. Die Verbesserung der Konstruktion für eine 400 W Hochdruck-Quecksilberdampflampe wird durch besseres quantitatives Verständnis der Transportvorgänge ermöglicht, wobei das Ziel die präzise Positionierung des Lichtbogens in der Mitte der Röhre ist, so daß die Isothermen parallel zur Wand verlaufen.

ИССЛЕДОВАНИЕ ТРЕХМЕРНОЙ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В РТУТНЫХ ЛАМПАХ ВЫСОКОГО ДАВЛЕНИЯ—I. ПАРАМЕТРИЧЕСКИЕ ИЗМЕНЕНИЯ ПРИ ГОРИЗОНТАЛЬНОМ РАСПОЛОЖЕНИИ

Аннотация—Для исследования естественной конвекции в горизонтальных паротутных дуговых лампах высокого давления построена трехмерная численная модель в криволинейных координатах и предложен эффективный метод расчета. При горизонтальном расположении дуги скорость конвекции направлена преимущественно поперек оси дуги. Возникающий изгиб разрядного столба вверху приводит к неравномерному нагреву кварцевой стенки, который может существенно ухудшить рабочие характеристики современной горизонтальной металлогалогенидной лампы. В результате проведенного исследования установлено влияние ряда конструктивных параметров (давления паров ртути, кривизны изогнутой дуговой лампы, расстояния между смещенными электродами, длины введения электродов) на температурное распределение в дуговой лампе. С качественной точки зрения вертикальное расположение изогнутой дуговой лампы может вызвать перемещение горячего ядра дуги снизу, а смещенные электроды могут привести к повышению температур вблизи концов дуговой лампы. Для дальнейшего улучшения конструкции ртутной лампы высокого давления мощностью 400 Вт посредством центрирования дуги и формирования изотерм, согласующихся с профилем стенки, необходимо иметь количественные характеристики процессов переноса.